Current driven magnetization dynamics in ferromagnetic nanowires with Dzyaloshinskii-Moriya interaction

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We study current induced magnetization dynamics in a long thin ferromagnetic wire with Dzyaloshinskii-Moriya interaction (DMI). We find a spiral domain wall configuration of the magnetization and obtain an analytical expression for the width of the domain wall as a function of the interaction strengths. Our findings show that above a certain value of DMI a domain wall configuration cannot exist in the wire. Below this value we determine the domain wall dynamics for small currents, and calculate the drift velocity of the domain wall along the wire. We show that the DMI suppresses the minimum value of current required to move the domain wall. Depending on its sign, the DMI increases or decreases the domain wall drift velocity.

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Introduction. A number of recent experiments, performed in various metallic magnets, have shown the spiral structure of magnetization due to Dzyaloshinskii-Moriya interaction (DMI) [1, 2, 3, 4, 5, 6, 7, 8]. In particular, the B20 structure of ferromagnets, such as MnSi, which lacks strict space-inversion symmetry, leads to a long-wavelength helical twist in the magnetization [4, 5, 6]. Furthermore, the direct space-time observation of the spiral structure by Lorentz microscopy became possible for DMI-induced helimagnets [7, 8]. Using spin-polarized scanning tunneling microscopy it has been shown that the magnetic order of 1 monolayer Mn on W(001) is a left-handed spiral [2] and that the magnetic structure of the Fe double layer on W(110) is a right-rotating spiral [1]. All these spiral states are the consequence of DMI.

A spin-polarized current flowing through such spiral magnetic structures would exert a spin-torque which could be used for manipulations of the magnetization with potential applications. For example, in magnetic memory devices [9, 10] the key issue is to manipulate the domain wall (DW) configurations by means of magnetic fields and/or spin-polarized current. Therefore, current-induced dynamics of spiral magnets is an important subject of technological relevance.

One of the most important factors which effects the DW motion is pinning. The DW pinning can have "extrinsic" and "intrinsic" nature. The extrinsic pinning is due to surface roughness and other irregularities of the wires which brake translational invariance. On the other hand, the intrinsic pinning is present even in ideally smooth (translation invariant) nanowires. It depends on the wire geometry and material parameters which can be described by anisotropies. Although extrinsic pinning can be significantly reduced in the near future with the help of more sophisticated wire fabrication techniques, the intrinsic pinning is always present. Therefore, in this Letter we concentrate on the more important case of DW dynamics with the intrinsic pinning.

We determine the effect of a polarized current on the magnetization configuration in the ferromagnetic wire with both strong easy-axis anisotropy along its axis and weak anisotropy in the plane transverse to the wire. The DMI, which arises from spin-orbit scattering of electrons in non-centrosymmetric magnetic materials is typically irrelevant in bulk metals as their crystals are inversion-symmetric. However, in low-dimensional systems (such as atomic layers and wires), which lack structural inversion symmetry, the DMI in the presence of softened ferromagnetic exchange coupling leads to the formation of the spiral spin structures.

The main goal of this Letter is to study the influence of DMI on the magnetization dynamics in ferromagnets. We obtain the expression for the DW width as a function of the DMI constant, uniaxial anisotropy along the wire, and exchange interaction constant. We find that there is a critical value of the DMI above which a DW configuration cannot exist in the wire. This result can have an important implication for the future experiments by setting a limit on the devices with DMI which use DWs for information manipulation. Below this critical value of DMI the DW can propagate along the wire and rotate around its axis. Any angle is equally favorable for the DW if there is no anisotropy in the transverse plane. Generally speaking in most wires there exists such an anisotropy due to the asymmetry of the wire cross section. We show that it leads to a chosen direction of the magnetization in the center of the DW, so that the wall cannot rotate freely anymore. Therefore, if a polarized current is passing through such a wire, the DW will move only if the current is larger than a certain critical value. This value corresponds to the minimal torque needed to be pumped into the system to rotate the spins of the DW around the wire's axis.

We investigate the dynamics of the DW in the small transverse anisotropy regime. In particular, we find the drift (average) velocity of the DW in the wire with DMI. Our findings also show that DMI decreases the critical value of current required to move a DW. To obtain all

these results for the DW dynamics, we use a universal method for finding zero mode dynamics of spin textures. This method is described in detail in the supplementary material [11].

Model. We employ a simple theoretical model of a ferromagnet with DMI and anisotropies which highlights a new kind of behavior of DW structures. We consider a Hamiltonian for a ferromagnet which has two terms describing the exchange and Dzyaloshinskii-Moriya interactions [12, 13]. Without anisotropies in the continuous limit this Hamiltonian takes the form,

$$\mathcal{H}_0 = \int d^3r \left[\frac{J_0}{2} (\nabla \mathbf{M})^2 + D_0 \mathbf{M} \cdot (\nabla \times \mathbf{M}) \right]. \quad (1)$$

Here $\mathbf{M}$ is a magnetization vector, $J_0 > 0$ is exchange interaction constant, and D_0 is the DMI constant. We study a ferromagnetic wire which is modeled as a one-dimensional (1D) classical spin chain [21], where the wire is along the z-axis, see Fig. 1. For the thin long wire with uniaxial anisotropy Hamiltonian (1) modifies to

$$\mathcal{H} = \int \mathrm{d}z \left[\frac{J}{2} (\partial \mathbf{S})^2 + D \mathbf{S} \cdot (\mathbf{e}_z \times \partial \mathbf{S}) - \lambda S_z^2 \right]. \quad (2)$$

Here $\mathbf{e}_z$ is the unit vector in z direction, $\partial = \partial/\partial z$, and we introduced normalized magnetization vector $\mathbf{S} = \mathbf{M}/M$, so that $\mathbf{S}^2 = 1$, $D = D_0/(AM^2)$, and $J = J_0/(AM^2)$, where A is the cross-sectional area of the wire. The last term in Eq. (2) is due to uniaxial anisotropy (with the anisotropy constant $\lambda = \lambda_0/(AM^2)$) which shows that the system favors the magnetization along the wire.

To study the magnetization dynamics we employ the generalized Landau-Lifshitz-Gilbert equation [14, 15] for 1D wire with current j along the wire:

$$\dot{\mathbf{S}} = \mathbf{S} \times \mathbf{H}_e - j\partial \mathbf{S} + \beta j \mathbf{S} \times \partial \mathbf{S} + \alpha \mathbf{S} \times \dot{\mathbf{S}}. \quad (3)$$

where $\mathbf{H}_e = \delta\mathcal{H}/\delta\mathbf{S}$, $\dot{\mathbf{S}} = d\mathbf{S}/dt$, $\alpha = \alpha_0/M^2$ and α_0 is the Gilbert damping constant, $\beta = \beta_0/M^2$ and β_0 is the constant of nonadiabatic current term, time is measured in the units of the gyromagnetic ratio $\gamma_0 = g\,|e|/(2mc)$, and the current j is measured in units of $a^3/(2eM\gamma_0)$ where a is the lattice constant. Generally speaking one also has to specify the boundary conditions for Eq. (3).

A general solution of one-dimensional LLG equation (3) can always be presented in the form

$$\partial \mathbf{S} = \Gamma(z,t) \mathbf{e}_z \times \mathbf{S} + \Lambda(z,t) \mathbf{S} \times [\mathbf{e}_z \times \mathbf{S}], \quad (4)$$

where Γ and Λ are in general two independent functions of z and t; it also follows that $\partial S_z = \Lambda(1 - S_z^2)$.

Zero current. First we consider the simplest case of zero current ($j = 0$) and look for a time-independent magnetization configuration. This means that we need to minimize Hamiltonian (2) which can be written up to a constant in the form

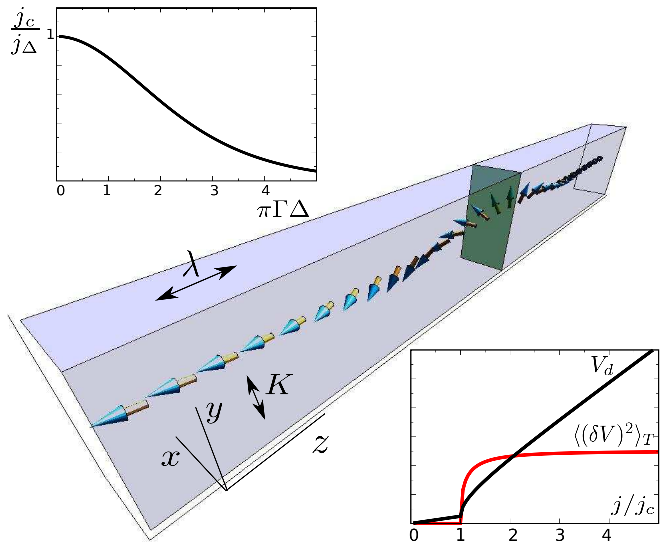

FIG. 1: (color online) Sketch of the wire with magnetization profile for a DW. λ and K denote the anisotropies along and transverse to the wire, respectively. The upper inset shows the dependence of j_c on the twist $\Gamma\Delta$, Eq. (18); the lower inset shows drift velocity V_d and variance $\langle(\delta V)^2\rangle_T$ (in arb. units) vs. current j, see Eqs. (20) and (21).

$$\mathcal{H} = \int \mathrm{d}z \left[\frac{J}{2} \left(\partial \mathbf{S} - \frac{D}{J} \mathbf{e}_z \times \mathbf{S} \right)^2 + \left(\lambda - \frac{D^2}{2J} \right) (1 - S_z^2) \right]. \quad (5)$$

The spin configuration depends on the sign of $\lambda - D^2/2J$.

For $2J\lambda < D^2$ the minimum of the second term is at $S_z = 0$. The first term is minimized by the condition $\partial \mathbf{S} = \frac{D}{J} \mathbf{e}_z \times \mathbf{S}$, so that the solution is a spiral,

$$\mathbf{S} = (\cos(\Gamma z + \phi_0), \sin(\Gamma z + \phi_0), 0)^T, \quad \Gamma = D/J. \quad (6)$$

The ground state is thus unique and there is no DW configuration. Therefore, for the wires with weak enough uniaxial anisotropy and/or exchange constant the spiral magnetization state can prevent the formation of DWs.

For $2J\lambda > D^2$ the minimum of the second term is at $S_z = \pm 1$. This also minimizes the first term in Eq. (5). Thus, $S_z = \pm 1$ are the two solutions, and a DW can exist in the wire as a transition from one solution to another.

Below we study the statics and dynamics of such a DW in the wire, and therefore we concentrate on the case $2J\lambda > D^2$. Then the boundary conditions for Eq. (3) are $S_z \to \pm 1$ at $z \to \pm\infty$. To find the static configuration of the DW we consider the solution in the form (4). Substituting it into Hamiltonian (5), we find

$$\mathcal{H} = \int \mathrm{d}z \left[\frac{J}{2} \left(\Gamma - \frac{D}{J} \right)^2 + \frac{J}{2} \Lambda^2 + \lambda - \frac{D^2}{2J} \right] (1 - S_z^2). \quad (7)$$

The minimization of the first term sets

$$\Gamma = D/J\,, \quad (8)$$

cf. Eq. (6). Using parametrization $S_z = \tanh f(z)$, we obtain

$$\mathcal{H} = \frac{J}{2}\int \mathrm{d}z \frac{(\partial f)^2 + \Delta^{-2}}{\cosh^2 f}\,, \quad \Delta^{-2} = \Delta_0^{-2} - \Gamma^2, \tag{9}$$

where $\Delta_0^2 = \sqrt{J/2\lambda}$ is the DW width in the absence of DMI. The straightforward minimization of Eq. (9) gives $f = z/\Delta$ or $\Lambda = 1/\Delta$, and in components the solution takes the form

$$S_x = \frac{\cos(\Gamma(z-z_0)+\phi)}{\cosh((z-z_0)/\Delta)}, \tag{10a}$$

$$S_y = \frac{\sin(\Gamma(z-z_0)+\phi)}{\cosh((z-z_0)/\Delta)}, \tag{10b}$$

$$S_z = \tanh(z-z_0)/\Delta), \tag{10c}$$

where the angle ϕ is the tilt of the DW, and z_0 is its position (both arbitrary). We see that $2\pi/\Gamma$ is the pitch of the spiral, Δ is the width of the DW and $\Gamma\Delta$ is thus the twist of the DW. Both Γ and Δ have the same functional dependencies in terms of J_0, D_0, and λ_0 as in terms of J, D, and λ. According to its definition in Eq. (9), Δ becomes infinite in the limit $2J\lambda = D^2$ and DW cannot be sustained in the wire.

The energy of the DW is $E = 2J/\Delta = 2\sqrt{2J\lambda - D^2}$. It vanishes when D^2 approaches $2J\lambda$.

The z component of the magnetization (10) is the same as that of a standard (without DMI) DW of width Δ. The direction of the twist of the DW depends on the sign of DMI and can be either clock or counterclockwise.

Parameters z_0 and ϕ in Eq. (10) correspond to two zero-modes of the system. These modes are the most relevant if the system is perturbed. The time-dependent solution then can be represented in the form of a moving and rotating DW plus a small correction to its shape. The requirement that the correction remains small during the motion leads to the equations for the velocity and angular velocity of the DW. A detailed derivation of these equations is presented in the supplementary material [11]. Below we present the results and discuss their implications.

Small currents. First we find the magnetization dynamics in the wire for small applied currents. We denote the solution (10) for the DW without a current, as $\mathbf{S}_0(z)$. When the current is applied we expect the DW to move and rotate. The full dynamics is described by the equation

$$\dot{\mathbf{S}} = \mathbf{S}\times\mathbf{H}_e + \mathbf{h}, \qquad \mathbf{H}_e = \delta\mathcal{H}/\delta\mathbf{S}, \tag{11}$$

where the correction $\mathbf{h}$ for small currents is $\mathbf{h} = \mathbf{h}_j$,

$$\mathbf{h}_j = -j\partial\mathbf{S}_0 + (\beta j - \alpha\dot{z}_0)\mathbf{S}_0\times\partial\mathbf{S}_0 + \alpha\dot{\phi}\mathbf{S}_0\times\mathbf{e}_z\times\mathbf{S}_0. \tag{12}$$

This correction gives the following results for the DW velocity and angular velocity:

$$\dot{z}_0 = \frac{1+\alpha\beta+(\alpha-\beta)\Gamma\Delta}{1+\alpha^2}j, \quad \dot{\phi} = \frac{(\alpha-\beta)\Delta}{(1+\alpha^2)\Delta_0^2}j\,. \tag{13}$$

A few conclusions can be made from these equations.

i.) The direction of the DW rotation depends only on the relative strength of the two dissipative terms in the LLG Eq. (3). Remarkably, the sign of the DMI correction to the DW velocity depends on weather the DW rotates in the same direction as the twist of the DW.

ii.) For $\beta = 0$ there is an integral of motion $\alpha(\Gamma^2 + \Delta^{-2})z_0 - (1/\Delta + \alpha\Gamma)\phi = \text{const}$. If we take $1/\Delta = 0$ which corresponds to a perfect spiral state (DW width is infinite), this invariant just describes the rotation of the spiral while it moves.

iii.) At very large twists $\Gamma\Delta$, $\dot{z}_0 = \dot{\phi}\Gamma J/2\lambda$, independently of both α and β.

iv.) The DW rotation and its velocity diverge when D^2 approaches $2\lambda J$. This nonphysical result is the consequence of the fact that our derivation of Eq. (13) neglects all modes except the zero ones. However, in the limit of $D^2 \to 2\lambda J$ the breathing mode (the mode that corresponds to the change of the DW width and pitch) softens and its dynamics cannot be neglected [16].

In line with the general result [17], in the special case of $\alpha = \beta$, the DW does not rotate and just moves with the velocity which depends on current only.

Small anisotropy in the transverse plane. In order to account for the anisotropy in the transverse plane we introduce a correction to Hamiltonian (2) in the form

$$\mathcal{H}_{xy} = \int \mathrm{d}z K S_y^2(z), \tag{14}$$

where the anisotropy constant $K > 0$ is very small.

The presence of this anisotropy fixes the tilt angle ϕ of the solution (10). To show it we calculate the correction to the energy to the first order in K by substituting Eq. (10) into Eq. (14). Assuming the wall to be at the origin ($z_0 = 0$), we obtain $\delta_1 E = K\Delta - \frac{2\pi K\Gamma\Delta^2}{\sinh(\pi\Gamma\Delta)}\cos(2\phi)$. This correction has a minimum at $\phi = 0, \pi$. When DMI is absent ($\Gamma = 0$) this correction reduces to $K\Delta[1 - 2\cos(2\phi)]$.

Dynamics and transverse anisotropy. Now we find how the small anisotropy in the transverse plane affects the magnetization dynamics. The correction $\mathbf{h}$ defined in Eq. (11) takes the form $\mathbf{h} = \mathbf{h}_j + \mathbf{h}_{xy}$, where $\mathbf{h}_j$ is given by Eq. (12) and

$$\mathbf{h}_{xy} = \mathbf{S}\times\frac{\delta\mathcal{H}_{xy}}{\delta\mathbf{S}} = 2KS_y\mathbf{S}\times\mathbf{e}_y\,. \tag{15}$$

This perturbation leads to the following equations for the position of the DW and the tilt angle:

$$\dot{z}_0 = \frac{\beta}{\alpha}j + \frac{(\alpha-\beta)(1+\alpha\Gamma\Delta)}{\alpha(1+\alpha^2)}\left[j - j_c\sin(2\phi)\right], \tag{16}$$

$$\dot{\phi} = \frac{(\alpha-\beta)\Delta}{(1+\alpha^2)\Delta_0^2}\left[j - j_c\sin(2\phi)\right], \tag{17}$$

where the critical current j_c is given by

$$j_c = j_\Delta\frac{\pi\Gamma\Delta}{\sinh(\pi\Gamma\Delta)}, \qquad j_\Delta = \frac{\alpha K\Delta}{|\alpha-\beta|}\,. \tag{18}$$

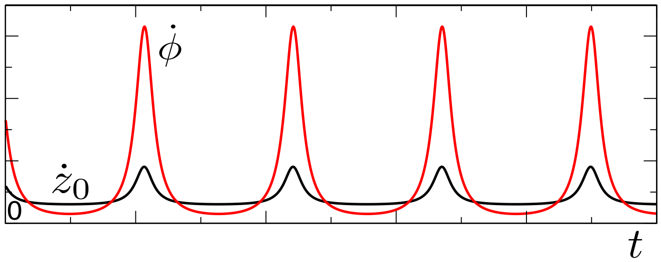

FIG. 2: (color online) Velocities $\dot{z}_0$ and $\dot{\phi}$ given by Eqs. (16) and (17) at $j = 1.1 j_c$ vs. time. (In arb. units)

j_Δ is a critical current for the domain wall of the same width, but without the twist. These equations reduce to Eq. (13) for $K = 0$. We also note that Eq. (18) is correct only in the first order in K.

The critical current j_c is *exponentially suppressed* for twists $\Gamma\Delta \gtrsim 1/\pi$. For small twists $j_c \approx j_\Delta(1 - \pi^2(\Gamma\Delta)^2/6)$. Note that j_c in Eq. (18) diverges at $\alpha = \beta$, that is the DW does not spin for any current [17].

For $j < j_c$ the DW tilts by the angle $\sin(2\phi_j) = j/j_c$ and moves with a constant velocity $\dot{z}_0 = j\beta/\alpha$, if $\beta = 0$, the DW does not move at all. For $j > j_c$ the DW both spins and moves along the wire.

Eqs. (17) and (16) can be solved analytically. The solution gives both the velocity and angular velocity, which periodically depend on time [18] (see Fig. 2), with the period T and average angular velocity Ω given by [11]:

$$\Omega = \frac{2\pi}{T} = \frac{(\alpha-\beta)\Delta}{(1+\alpha^2)\Delta_0^2}\sqrt{j^2 - j_c^2}. \tag{19}$$

More experimentally relevant, however, is the average (drift) velocity of the DW $V_d = \langle \dot{z}_0 \rangle_T$. For any current it is given by [11]

$$V_d = \begin{cases} \frac{\beta}{\alpha} j, & \text{for } j < j_c, \\ \frac{\beta}{\alpha} j + \frac{(\alpha-\beta)(1+\alpha\Gamma\Delta)}{\alpha(1+\alpha^2)}\sqrt{j^2 - j_c^2}, & \text{for } j > j_c. \end{cases} \tag{20}$$

The square of the deviation of the velocity from the drift velocity, Eq. (20), $\langle (\delta V)^2 \rangle_T$ is

$$\langle (\delta V)^2 \rangle_T = \begin{cases} 0, & \text{for } j < j_c, \\ \left[\frac{(\alpha-\beta)(1+\alpha\Gamma\Delta)}{\alpha(1+\alpha^2)} j_c\right]^2 \frac{\sqrt{j^2-j_c^2}}{j+\sqrt{j^2-j_c^2}}, & \text{for } j > j_c. \end{cases} \tag{21}$$

Both $V_d(j)$ and $\langle (\delta V)^2 \rangle_T$ are shown in the inset of Fig. 1. For large currents, $j \gg j_c$, the drift velocity asymptotically approaches the velocity given by Eq. (13), while $\langle (\delta V)^2 \rangle_T$ approaches a constant.

Summary. We have studied the effects of DMI on the magnetization statics and dynamics in a thin ferromagnetic wire. We have derived a simple criterion which determines whether the wire with the spiral magnetization state can sustain a DW configuration. Namely, in the wires with weak enough uniaxial anisotropy and/or exchange constant compared to DMI constant ($2J\lambda < D^2$) a DW cannot be formed. In the opposite case ($2J\lambda > D^2$) we have found the spiral magnetization state with a DW in the wire. For $\beta = 0$ the wall moves along the wire only if the applied current is above j_c given by Eq. (18). The variance of the velocity in this regime is given by $\langle (\delta V)^2 \rangle_T = V_d^2(j/\sqrt{j^2 - j_c^2} - 1)$. For $\beta \neq 0$ the DW moves but does not rotate for currents below j_c. Above j_c the DW both moves and rotates [18]. Our result, Eq. (18), shows that the critical value of current is suppressed by DMI. We also have derived the expression, Eq. (20), for the drift velocity V_d of the DW for all values of current. It shows that above the critical current j_c the drift velocity can be enhanced by DMI.

We believe that our findings can be experimentally observed, e.g., with the use of the scanning tunneling microscopy which was employed to reveal the DW structure in ultrathin Fe nanowires [19, 20]. We note that in a realistic experimental setting besides the "intrinsic" pinning there always going to be an extrinsic pinning due to a nonideal shape of the wire. It is, however, clear that in the near future the development of better nanofabrication techniques will lead to the situation when one has to worry mostly about the "intrinsic" effect.

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